

Group Assignment #4 (Week of Feb 4, 2002)

1. Write a program that takes a list of ordered pairs and multiplies all the first numbers in each pair together. Example: $p1 [(2,4), (6,8)] = 12$

```
p10 = foldr (*) 1 . map pickfirst
      where
      pickfirst (x,y) = x
```

2. The list of the students in grade 3 to grade 5 is as follows:
student_rel = [(5, "Bob"),(3, "Chris"),(3, "Toni"),(4, "Dave"),
 (4, "Bruce"), (5, "Tom")]

Write a recursive program that takes, as input, a grade number and the student list and returns a list of all the students in that grade.

Example: find_student 4 student_rel = ["Dave", "Bruce"]

```
find_student x [ ] = [ ]
find_student x ((a,b):as) = b:find_student x as, if x = a
                          = find_student x as, otherwise
```

3. The list of the students' average in grade 3 to grade 5 is as follow:
student_avg = [(5, "Bob",80),(3, "Chris",95),(3, "Toni",80),
 (4, "Dave",76),(4, "Bruce",65), (5, "Tom",96)]

Write a recursive program to find the highest average in the list.

```
p3 [ ] = 0
p3 ((a,b,c):as) = c, if c >= p3 as
                = p3 as, otherwise
```

4. Write a program that returns the highest average in the list above (i.e., in student_avg) using list comprehension.

```
p4 avg_list = highest [c|(a,b,c)<- avg_list]
highest [ ] = 0
highest (e:es) = e, if e >= highest es
               = highest es, otherwise
```

5. Prove by mathematical induction that
 $3 + 9 + 15 + \dots + (6n - 3) = 3n^2$
is true for every positive integer n.

Base Case: $n=1$

$$\text{Lhs} = (6 \cdot 1 - 3) = 3 \quad \text{Rhs} = 3 \quad \text{LHS} = \text{RHS}$$

Assume hypothesis:

$$3 + 9 + 15 + \dots + (6k - 3) = 3k^2$$

$$\text{Show: } 3 + 9 + 15 + \dots + (6k - 3) + (6(k+1) - 3) = 3(k+1)^2$$

$$\begin{aligned} \text{Lhs} &= 3 + 9 + 15 + \dots + (6k - 3) + (6(k+1) - 3) \\ &= 3k^2 + (6(k+1) - 3) && \textit{ex hypothesis} \\ &= 3k^2 + 6k + 3 \\ &= 3(k^2 + 2k + 1) && \textit{factoring} \\ &= 3(k+1)^2 && \textit{quadratic roots} \\ &= \text{RHS} \end{aligned}$$

Therefore, by induction on 'n', $3 + 9 + 15 + \dots + (6n - 3) = 3n^2$ ■