60-100 Assignment # 2 (Due 28th March, 2002)

1. (a) State formally when a relation is reflexive, symmetric and transitive.
(b) An equivalence relation is one which is reflexive, symmetric, and transitive. A classic problem that illustrates the concept of equivalence is the relationship between spare parts produced by different manufacturers. Consider a group of "hypothetical" manufacturers called *Titan*, *Orion*, and *Ursus*, specializing in the production of some spare parts. We could represent them as:

Parts = {Titan, Orion, Ursus}

Let the parts each of these produces be:

Titan = $\{T_1, T_2, T_3, T_4\}$
Orion = $\{O_1, O_2, O_3, O_4, O_5\}$
Ursus = $\{U_1, U_2, U_3\}$

Let the following also be true:

i. The parts T_1, T_3, O_1, and O_3 may be used as substitutes for each other.
ii. The parts T_4 and O_5 may be used as substitutes for each other.
iii. The parts U_1, U_2 and O_4 may be used as substitutes for each other.
iv. The parts T_2, O_2, and U_3 each have no other substitutes other than themselves.

Given this information derive the following: sub_ref (the set of all reflexive pairs), sub_sym (the set of all symmetric pairs), and sub_trans (the set of all transitive pairs). All of these relations must satisfy the constraints above. Combine all these relations to create the set *substitutes* which is an equivalence relation.

A strict reading of the constraints above should reveal symmetries involved (i.e., the constraints do in fact form the set of all the symmetric pairs!).

sub_sym = {(T1,O1), (O1,T1),(O1,O3), (O3, O1), (T1,T3), (T3,T1), (T1,O3), (O3,T1), (O1,T3), (T3,O1), (T3,O3),(O3,T3), (T4,O5), (O5, T4), (U1,U2),(U2,U1), (U1,O4), (O4,U1), (U2,O4),(O4,U2), (T2,T2), (O2,O2), (U3,U3)}

Adding all the reflexive pairs and all the transitive pairs, we have:

Substitutes = {(T1,O1), (O1,T1),(O1,O3), (O3, O1), (T1,T3), (T3,T1), (T1,O3), (O3,T1), (O1,T3), (T3,O1), (T3,O3),(O3,T3), (T4,O5), (O5, T4), (U1,U2),(U2,U1), (U1,O4), (O4,U1), (U2,O4),(O4,U2), (T1,T1),(T2,T2), (T3,T3), (T4,T4), (O1,O1),(O2,O2), (O3,O3),(O4,O4), (O5,O5), (U1,U1), (U2,U2), (U3,U3)}

2. Write a grammar that generates the set $\{0^m1^n \mid m, n \text{ are nonnegative integers}\}$

There are at least two grammars for this set (I'll assume it is non-empty).

$G_1 = (T, N, S, P)$ where

$T = \{0,1\}$
$N = \{ S \}$
$S = S$
$P = \{ S \rightarrow 0S, S \rightarrow S1, S \rightarrow \lambda\}$

(Note: λ is an empty string. Alternatively instead of $S \rightarrow \lambda$, substitute $S \rightarrow 1$ and, on first glance, this should work too.)

$G_2 = (T, N, S, P)$ where

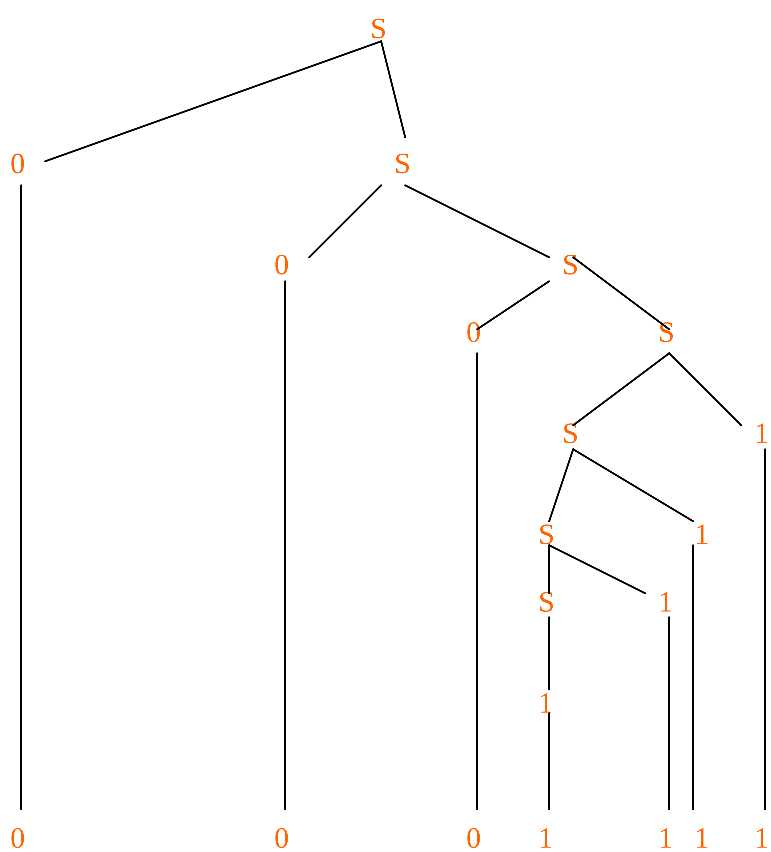
$T = \{0,1\}$
$N = \{ S, A \}$
$S = S$
$P = \{ S \rightarrow 0S, S \rightarrow 1A, S \rightarrow 1, A \rightarrow 1A, A \rightarrow 1, S \rightarrow \lambda\}$

3. (a) From the grammar in (2) construct a derivation of 0^21^4 (i.e. (001111)
(b) Using the grammar in (2), show a syntax tree of 0^31^4

(a) Using G_1:

$S \Rightarrow 0S \Rightarrow 00S \Rightarrow 00S1 \Rightarrow 00S11 \Rightarrow 00S111 \Rightarrow 00S1111 \Rightarrow 001111$

(b)



4. || These are the flights of some airlines. Each record lists the
||flight number, origin, destiny, departure time, arrival time, and price of the ticket

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Canada_rel = [(“CA101”, “Toronto”,  “Vancouver”, 1200, 1700, 400),
              (“CA121”, “Toronto”,  “Ottawa”, 700, 800, 200),
              (“CA220”, “Montreal”, “Halifax”, 800, 1400, 400),
              (“CA330”, “Vancouver”, “Hong Kong”, 1300, 1900, 1000)]

Cathy_rel = [(“CP001”, “Hong Kong”, “Vancouver”, 1500, 2330, 800),
             (“CP300”, “Vancouver”, “Hong Kong”, 900, 1500, 900),
             (“CP322”, “Hong Kong”, “Sydney”, 700, 1400, 600),
             (“CP330”, “Sydney”, “Hong Kong”, 900, 1600, 600)]

Australia_rel = [(“A112”, “Sydney”, “Hong Kong”, 1200, 1900, 500),
                 (“A213”, “Hong Kong”, “London”, 900, 1500, 800),
                 (“A220”, “Sydney”, “Jakarta”, 730, 1230, 400),
                 (“A310”, “Sydney”, “Perth”, 800, 1000, 200)]

airline_rel = [canada_rel, cathy_rel, australia_rel]
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a. Write a function to output all possible destinations of Australian Airline flight originated from Sydney.

b. Write a function to output all possible destinations of any airline flying out from Hong Kong

c. Write a program that will output the possible flight(s) to take according to the preferences of the customer. The customer is allowed to get the quote of a similar flight in all airlines.

For example, the customer wants to go from Toronto to Vancouver with a direct flight of Canadian Airline that will not cost more than $600.

Journey (“Canadian”, “Toronto”, “Vancouver”, 600, “Direct”)
= [(“CA101”, “Toronto”, Vancouver”, 1200, 1700, 400)]

If the customer wants to go from Sydney to Hong Kong with a direct flight of any Airline that will not cost more than $900

Journey (“Any”, “Sydney”, “Hong Kong”, 900, “Direct”)
= [(“CP330”, “Sydney”, “Hong Kong, 900, 1600, 600), (“A112”, “Sydney”, “Hong Kong”, 1200,1900,500)]

(Hint: You need to write some functions to be used along with join, select, and project)

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Select_2_3_6_of_6 cond1 cond2 cond3 rel = [(a,b,c,d,e,f)|(a,b,c,d,e,f)<-rel; b= cond1;
                                           c= cond2; f < cond3]

journey (a,b,c,d,e) = select_flight (a,b,c,d), if e = “direct”
                    = join_journey (a,b,c,d), otherwise

join_journey (a,b,c,d) = []
select_from_all b c d [] = []
select_from_all b c d (e:es) = select_2_3_6_of_6 b c d e ++
                               select_from_all b c d es

select_flight (a,b,c,d)
      = select_from_all b c d airline_rel, if a = "Any"
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          = select_2_3_6_of_6 b c d canada_rel, if a = "Canadian"
          = select_2_3_6_of_6 b c d australia_rel, if a = "Australia"
          = select_2_3_6_of_6 b c d cathy_rel, if a = "Cathy"
          = [ ], otherwise

|| Journey is a function that takes the following information as input
|| What airline the customer prefer?
|| Origin and destination
|| What price range?
|| Flying direct only?
journey (a,b,c,d,e) = select_flight (a,b,c,d), if e = "Direct"
                    = [ ], otherwise
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5. Show by (mathematical or structural) induction:

(a) $[(p_1 \rightarrow p_2) \wedge (p_2 \rightarrow p_3) \wedge \ldots \wedge (p_{n-1} \rightarrow p_n)] \rightarrow [(p_1 \wedge p_2 \wedge \ldots \wedge p_{n-1}) \rightarrow p_n]$ is always true.

BASE CASE: $(p_1 \rightarrow p_2) \rightarrow (p_1 \rightarrow p_2)$ is a tautology

ASSUME (inductive hypothesis) $[(p_1 \rightarrow p_2) \wedge (p_2 \rightarrow p_3) \wedge \ldots \wedge (p_{n-1} \rightarrow p_n)] \rightarrow [(p_1 \wedge p_2 \wedge \ldots \wedge p_{n-1}) \rightarrow p_n]$

Show: $[(p_1 \rightarrow p_2) \wedge (p_2 \rightarrow p_3) \wedge \ldots \wedge (p_{n-1} \rightarrow p_n) \wedge (p_n \rightarrow p_{n+1})] \rightarrow [(p_1 \wedge p_2 \wedge \ldots \wedge p_{n-1} \wedge p_n) \rightarrow p_{n+1}]$

Assume the antecedent of this implication is true, namely $[(p_1 \rightarrow p_2) \wedge (p_2 \rightarrow p_3) \wedge \ldots \wedge (p_{n-1} \rightarrow p_n) \wedge (p_n \rightarrow p_{n+1})]$ is true and the consequent is false, namely $[(p_1 \wedge p_2 \wedge \ldots \wedge p_{n-1} \wedge p_n) \rightarrow p_{n+1}]$.

By the hypothesis, the first n-1 conjuncts of the antecedent are true, so $p_n \rightarrow p_{n+1}$ must also be true. Given that p_n is true (ex hypothesi), p_{n+1} must also be true.

The consequent is assumed to be false, however with p_{n+1} being true, there is no way to make $[(p_1 \wedge p_2 \wedge \ldots \wedge p_{n-1} \wedge p_n) \rightarrow p_{n+1}]$ false and meet this assumption.

Therefore, by induction,

$[(p_1 \rightarrow p_2) \wedge (p_2 \rightarrow p_3) \wedge \ldots \wedge (p_{n-1} \rightarrow p_n)] \rightarrow [(p_1 \wedge p_2 \wedge \ldots \wedge p_{n-1}) \rightarrow p_n]$ is always true. ■

(b) Prove that every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps.

Base Case: Postage of 12 cents can be formed using three 4-cent stamps.

Assume inductive hypothesis: that every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps. (i.e., postage of *n* cents can be formed using 4-cent and 5-cent stamps)

Show: that postage of n+1 cents can be formed using 4-cent and 5-cent stamps.

There are two cases to consider in this solution.

Case 1: At least one 4-cent stamp was used to form postage of n-cents.

In this case replace one 4-cent stamp with a 5-cent stamp to form postage of n+1 cents.

Case 2: No 4-cent stamps were used to form postage of n-cents.

That is, only 5-cent stamps were used. Since $n \geq 12$, at least three 5-cent stamps had to be used. So replace any three 5-cent stamps with four 4-cent stamps to form postage of n+1 cents.

Therefore, by induction, every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps.

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