# 60-100 Assignment # 2 (Due 28th March, 2002)

1. (a) State formally when is a relation is reflexive, symmetric and transitive.
(b) An equivalence relation is one which is reflexive, symmetric, and transitive. A classic problem that illustrates the concept of equivalence is the relationship between spare parts produced by different manufacturers. Consider a group of "hypothetical" manufacturers called *Titan*, *Orion*, and *Ursus*, specializing in the production of some spare parts. We could represent them as:

*Parts* = {Titan, Orion, Ursus}

Let the parts each of these produces be:

*Titan* = $\{T_1,T_2,T_3,T_4\}$
*Orion* = $\{O_1,O_2, O_3,O_4,O_5\}$
*Ursus* = $\{U_1,U_2,U_3\}$

Let the following also be true:

i. The parts $T_1$, $T_3$, $O_1$, and $O_3$ may be used as substitutes for each other.
ii. The parts $T_4$ and $O_5$ may be used as substitutes for each other.
iii. The parts $U_1$,$U_2$ and $O_4$ may be used as substitutes for each other.
iv. The parts $T_2$, $O_2$, and $U_3$ each have no other substitutes other than themselves.

Given this information derive the following: sub_ref (the set of all reflexive pairs), sub_sym (the set of all symmetric pairs), and sub_trans (the set of all transitive pairs). All of these relations must satisfy the constraints above. Combine all these relations to create the set *substitutes* which is an equivalence relation.

2. Write a grammar that generates the set $\{0^m1^n \mid m, n$ are nonnegative integers$\}$

3. (a) From the grammar in (2) construct a derivation of $0^21^4$ (i.e. (001111))
(b) Using the grammar in (2), show a syntax tree of $0^31^4$

4. || These are the flights of some airlines. Each record lists the
||flight number, origin, destiny, departure time, arrival time, and price of the ticket

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Canada_rel = [("CA101", "Toronto",  "Vancouver", 1200, 1700, 400),
              ("CA121", "Toronto",  "Ottawa", 700, 800, 200),
              ("CA220", "Montreal", "Halifax", 800, 1400, 400),
              ("CA330", "Vancouver", "Hong Kong", 1300, 1900, 1000)]

Cathy_rel = [("CP001", "Hong Kong", "Vancouver", 1500, 2330, 800),
             ("CP300", "Vancouver", "Hong Kong", 900, 1500, 900),
             ("CP322", "Hong Kong", "Sydney", 700, 1400, 600),
             ("CP330", "Sydney", "Hong Kong", 900, 1600, 600)]

Australia_rel = [("A112", "Sydney", "Hong Kong", 1200, 1900, 500),
                 ("A213", "Hong Kong", "London", 900, 1500, 800),
                 ("A220", "Sydney", "Jakarta", 730, 1230, 400),
                 ("A310", "Sydney", "Perth", 800, 1000, 200)]

airline_rel = [canada_rel, cathy_rel, australia_rel]
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a. Write a function to output all possible destinations of Australian Airline flight originated from Sydney.

b. Write a function to output all possible destinations of any airline flying out from Hong Kong

c. Write a program that will output the possible flight(s) to take according to the preferences of the customer. The customer is allowed to get the quote of a similar flight in all airlines.

For example, the customer wants to go from Toronto to Vancouver with a direct flight of Canadian Airline that will not cost more than $600.

Journey (“Canadian”, “Toronto”, “Vancouver”, 600, “Direct”)
= [(“CA101”, “Toronto”, Vancouver”, 1200, 1700, 400)]

If the customer wants to go from Sydney to Hong Kong with a direct flight of any Airline that will not cost more than $900

Journey (“Any”, “Sydney”, “Hong Kong”, 900, “Direct”)
= [(“CP330”, “Sydney”, “Hong Kong, 900, 1600, 600), (“A112”, “Sydney”, “Hong Kong”, 1200,1900,500)]

(Hint: You need to write some functions to be used along with join, select, and project)

5. Show by (mathematical or structural) induction:

(a) $[(p_1 \rightarrow p_2) \wedge (p_2 \rightarrow p_3) \wedge \ldots \wedge (p_{n-1} \rightarrow p_n)] \rightarrow [(p_1 \wedge p_2 \wedge \ldots \wedge p_{n-1}) \rightarrow p_n]$ is always true.

(b) Prove that every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps.