V. Conclusions

The Tarmy–Ghausi perturbation expressions have been successfully extended from the fifth to the general order with formulation in both combinatorial and recursive forms. Since $\tilde{T}$ and $\tilde{Q}$ are functions of each other in (3) and (4), then evaluation can proceed either by solving simultaneously for $\tilde{T}$ and $\tilde{Q}$ or by assuming small perturbations. The method can be applied to active-RC and active-R filters with any number of linear parasitics.

The technique offers several facilities which are not available by direct numerical evaluation:

(a) optimization of topology and design by analytical insight;
(b) predistortion;
(c) unlimited accuracy;
(d) a fourfold saving in computer execution time compared with the root-finding method adopted;
(e) amenability of the algebraic approach to evaluation of large-signal nonlinear perturbations.

REFERENCES


Design of Linear Phase Circularly Symmetric Two-Dimensional Recursive Digital Filters

HON KEUNG KWAN AND CHUN LEUNG CHAN

Abstract — A new design method for the design of two-dimensional circularly symmetric recursive digital filters possessing linear phase characteristics and approximately equiripple magnitude response is presented. The filter format is octagonally symmetric and is inherently stable. The denominator and the numerator of the transfer function are designed separately. The denominator is used to approximate the linear phase specifications while the numerator is used to approximate the overall magnitude characteristics. This method is suitable for both low-pass and bandpass filters design. The computation involved is simple and requires short computational time. The resultant filter format is also suitable for modular implementation.

I. INTRODUCTION

Two-dimensional (2-D) digital filters are finding wide applications in many areas, such as image processing, magnetic data processing, ultrasonic data processing and biomedical tomography. Most of these applications involve images. In processing images, the requirement of linear phase is practical important [1].

In the last decade, some design methods [2]–[6] for the 2-D circularly symmetric recursive digital filters which satisfy both magnitude and constant group delay characteristics have been introduced. Recently, Hinamoto and Maekawa [7] has introduced an effective method in which the denominator and the numerator of the transfer function are designed separately. It has the advantages of reduction of the amount of calculations, improvement of convergence and stable structure. However, it still requires relatively complex computation and long computational times. In [8], a design method which simplifies this method by using the octagonal symmetry properties of the filter transfer function has been introduced.

In this paper, we are going to introduce a new method for the design of 2-D recursive digital filters which possess maximally linear phase characteristics and approximately equiripple magnitude response. This method also designs the numerator and the denominator of the transfer function separately and consists of two steps. The first step is to design a separable all-pole 2-D denominator which is formed by cascading two one-dimensional (1-D) filters expressed in the form of the hypergeometric series [9] and has a maximally linear phase characteristic. The second step is to design a zero-phase (or linear phase (see Section IX))
non-recursive 2-D filter to meet the overall magnitude specifications when combined with the previously determined 2-D denominator. This step can be carried out by employing the 1-D to 2-D modified McClellan transformation [10] (or a nonlinear optimization technique (see Section IX)).

II. FILTER FORMAT

The transfer function of the filter is of the following format:

\[
H(z^{-1}_1, z^{-1}_2) = \frac{N(z^{-1}_1, z^{-1}_2)}{D(z^{-1}_1) D(z^{-1}_2)}
\]

The denominator of \( H \) is separable and possesses maximally linear phase and circularly symmetric magnitude responses. The numerator \( N(z^{-1}_1, z^{-1}_2) \) possesses zero phase and circularly symmetric magnitude characteristics. Hence, the resulting filter \( H(z^{-1}_1, z^{-1}_2) \) is of linear phase and also circularly symmetric in magnitude response. The denominator and the numerator are designed separately to meet, respectively, the prescribed linear phase and magnitude response specifications.

III. ALL-POLE LINEAR PHASE DIGITAL FILTER DESIGN

With reference to \( [11, \text{eq. (8)}] \), a 1-D all-pole digital filter (formed from hypergeometric series [9]) possessing maximally flat group delay or maximally linear phase response can be expressed
where $N$ is the filter order, $\tau$ is the desired group delay, $z = \exp(j\omega)$ and $w$ is the normalized digital frequency in rad/s.

According to the Figs. 2 and 3, [11, sect. IV], and our personal experience, the filter (2) has the following properties:

(a) The higher the filter order $N$ is, the higher will be the cutoff frequency.

(b) The higher the filter order $N$ is, the larger will be the frequency range possessing maximally linear phase response.

(c) The greater the desired group delay is, the lower will be the cutoff frequency.

(d) The stability of the transfer function is guaranteed for all finite positive values of group delay.

A 2-D separable all-pole digital filter can be formed by cascading two identical 1-D filters as given in (2) in both $z_1^{-1}$ and $z_2^{-1}$ domains ($z_1 = \exp(j\omega_1)$ and $z_2 = \exp(j\omega_2)$) as

$$\frac{1}{D(z_1^{-1}, z_2^{-1})} = \frac{1}{D(z_1^{-1})} \cdot \frac{1}{D(z_2^{-1})}. \tag{3}$$

Assuming that the group delays of $1/D(z_i^{-1})$ for $i = 1, 2$ in the directions of $\omega_1$ and $\omega_2$ are $\tau_1$ and $\tau_2$, respectively and are given to be maximally flat. Therefore, the phase response of $1/D(z_1^{-1} z_2^{-1})$, which is the sum of the phases of $1/D(z_1^{-1})$ and that of $1/D(z_2^{-1})$, must also be maximally linear. Furthermore, it is interesting to find that the magnitude response of (3) is almost circular symmetric. Fig. 1(a)–(c) show the magnitude and phase responses of a 2-D all-pole filter with $N = 10$ and $\tau = \tau_1 = \tau_2 = 4$.

IV. 1-D NON-RECURSIVE DIGITAL FILTER DESIGN

After the design of the denominator, the next step is to design a 1-D non-recursive digital filter of the following format:

$$N(z^{-1}) = \sum_{m=0}^{M} h(m) \cos(mw) \tag{4}$$

where

$$\cos(mw) = (z^{-m} + z^{m})/2. \tag{5}$$

This non-recursive filter has a zero group delay. The coefficients $h(m)$, $m = 0, 1, 2, \ldots, M$, are to be obtained by optimization using the Remez exchange method [12] in an equiripple sense. Supposing that $H_1(z_1^{-1}, z_2^{-1})$ is the desired magnitude response at $(\omega_1, \omega_2)$, $H_2(z_1^{-1}, 0)D(z_2^{-1}, 0)$ will be used as the desired magnitude of $N(z^{-1})$.

V. 2-D NON-RECURSIVE DIGITAL FILTER DESIGN

By means of the modified McClellan transformation, obtained from [10, eq. (36)], the desired 2-D circularly symmetric non-recursive digital filter can be obtained from (4) as

$$N(z_1^{-1}, z_2^{-1}) = \sum_{m=0}^{M} h(m) \cos(mw) |_{\cos(mw) = (z_1^{-m} + z_1^{m})/2; \cos(mw) = (z_2^{-m} + z_2^{m})/2} \tag{6}$$

where

$$f(mw_1, mw_2) = 2[\cos^2(mw_1/2) \cos^2(mw_2/2)
- \sin^2(mw_1/2) \sin^2(mw_2/2)] - 1. \tag{7}$$

The constant $\alpha$ is chosen such that best circularity can be obtained. It can be searched out easily by a single variable iteration.

VI. OVERALL 2-D RECURSIVE DIGITAL FILTER DESIGN

Cascading the 2-D all-pole digital filter designed in the Section III with the 2-D non-recursive digital filter designed in the Sections IV and V, a 2-D recursive digital filter $H(z_1^{-1}, z_1^{-1})$ is obtained. According to [11, sect. IV] the 1-D recursive filters $1/D(z^{-1})$ for $i = 1, 2$ are stable for all positive values of $\tau$. Therefore, the resultant 2-D digital filter is also stable for all finite positive values of $\tau_1$ and $\tau_2$.

VII. MODULAR IMPLEMENTATION

In general, (2) can be factorized into first- and second-order polynomials in $z^{-1}$. Therefore, the denominator of the $N(z_1^{-1}, z_2^{-1})$ can easily be expressed into a product of first- and second-order sections as

For even $N$,

$$1/D(z_1^{-1}, z_2^{-1}) = A \prod_{i=1}^{N/2} \frac{1}{(1 + a_{i1}z_1^{-1} + a_{i2}z_2^{-1})(1 + a_{i1}z_1^{-1} + a_{i2}z_2^{-1})} \tag{8}$$

and for odd $N$,

$$1/D(z_1^{-1}, z_2^{-1}) = A \prod_{i=1}^{(N-1)/2} \frac{1}{(1 + a_{i1}z_1^{-1} + a_{i2}z_2^{-1})(1 + a_{i1}z_1^{-1} + a_{i2}z_2^{-1})} \tag{9}$$

Similarly, (4) can be factorized into first- and second-order polynomials in $\cos w$. Hence, for even $M$,

$$N(z^{-1}) = \sum_{i=1}^{M/2} \left( b_{i1} \cos^2(w) + b_{i2} \cos(w) + b_{i0} \right) \tag{10}$$

and for odd $M$,

$$N(z^{-1}) = (b_{00} \cos(w) + b_{0}) \cdot \prod_{i=1}^{(M-1)/2} \left( b_{i1} \cos^2(w) + b_{i2} \cos(w) + b_{i0} \right) \tag{11}$$

By using (7), (10), and (11) can be transformed into a 2-D circularly symmetric non-recursive digital filter. On substituting the following identities:

$$\cos(w_1) = (z_1 + z_1^{-1})/2 \tag{12}$$

$$\cos(w_2) = (z_2 + z_2^{-1})/2. \tag{13}$$

Each of the resulting 2-D equations of (10) and (11) is represented in terms of a product of second- and fourth-order sections of $z_1^{-1}$ and $z_2^{-1}$. Hence, the resultant $H(z_1^{-1}, z_1^{-1})$ can be implemented by a cascade of second- and fourth-order 2-D recursive filter modules.
VIII. DESIGN EXAMPLES

In each of the two examples given below, discrete frequency points are taken at intervals of 0.1π on the \(w_1\) axis within the passband radius (for the low-pass filter or up to the upper passband radius for the bandpass filter) for the computation of the relative root mean square (RMS) errors in the group delay as given in [8, eq. (26)]. Moreover, discrete frequency points are also taken at intervals of 0.1π in the region \(R\) defined in the eqn. (22) of [8] for the computation of the RMS errors in the magnitude response as given in [8, eq. (27)].

A. Example 1: Low-Pass Filter

A 20th-order 2-D circularly symmetric low-pass recursive digital filter with the following specification is designed.

\[
H_d(z_1^{-1}, z_2^{-1}) = \begin{cases} 
1.0, & \text{for } 0.0 \leq r \leq 0.5 \pi \text{ rad/s} \\
0.0, & \text{for } 0.7 \pi \leq r \leq \pi \text{ rad/s}
\end{cases}
\]

where \(r = \sqrt{(w_1^2 + w_2^2)}\). The group delays in both \(w_1\) and \(w_2\) directions, \(\tau_1\) and \(\tau_2\), are required to be \(2\) in the passband where \(r \leq 0.5 \pi \text{ rad/s}\). The resultant digital filter transfer function is obtained as

\[
H(z_1^{-1}, z_2^{-1}) = 9.7794 E - 10
\]

The magnitude and phase responses of the filter are shown in Fig. 2(a)–(c). According to the given specifications of this example, the parameters of the RMS error in group delay as given in [8, eq. (26)] are given by \(\tau = \tau_1 = \tau_2 = 2\), \(M^* = 6\), \(L = 5\), \(m_1 = 4\). Moreover, by defining the group delay weighting functions, \(u_m\), to be all equal to \(1\) for \(g = 1–6\), the group delay RMS error of this filter using the present design method is \(1.03 \times 10^{-3}\). Similarly, the parameter of the RMS error in magnitude response as given by [8, eq. (27)] is given by \(M^* = 66\). Moreover, by defining the magnitude response all-weighting functions \(u_m\) to be all equal to \(1\) at the discrete frequency points, as specified by the [8, eq. (22) or Table I] the magnitude response RMS error of this filter using the present method is \(2.8\). On designing a 20th-order filter with the same specifications and the same set of parameters as given above using the method of [8], the corresponding group delay and magnitude response RMS errors are, respectively, \(2.9 \times 10^{-12}\) and \(3.3\). The group delay error of the present method is larger than that of the method of [8]. However, the actual error is only \(1.03 \times 10^{-3}\) which is already very small by itself. On the other hand, the magnitude response error of the present method is slightly better than that of the method [8].

### Table I

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\[(1 - 0.3178 z_1^{-1} + 0.1380 z_2^{-1})(1 - 0.2124 z_1^{-1} + 0.1818 z_2^{-1})(1 - 0.3178 z_1^{-1} + 0.1380 z_2^{-1})(1 - 0.2124 z_1^{-1} + 0.1818 z_2^{-1})\]

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\[(1 - 0.4003 z_1^{-1} + 0.1130 z_2^{-1})(1 - 0.4497 z_1^{-1} + 0.07268 z_2^{-1})(1 - 0.4003 z_1^{-1} + 0.1130 z_2^{-1})(1 - 0.4497 z_1^{-1} + 0.07268 z_2^{-1})\]

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\[(1 - 0.04555 z_1^{-1} + 0.2604 z_2^{-1})(1 + 0.2522 z_1^{-1} + 0.4365 z_2^{-1})(1 - 0.04555 z_1^{-1} + 0.2604 z_2^{-1})(1 + 0.2522 z_1^{-1} + 0.4365 z_2^{-1})\]

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\[(1 - 0.4388 z_1^{-1} + 0.09620 z_2^{-1})(1 - 0.5539 z_1^{-1} + 0.07464 z_2^{-1})(1 - 0.4388 z_1^{-1} + 0.09620 z_2^{-1})(1 - 0.5539 z_1^{-1} + 0.07464 z_2^{-1})\]

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\[(1 - 0.5742 z_1^{-1} + 0.1012 z_2^{-1})(1 - 0.4646 z_1^{-1} + 0.05917 z_2^{-1})(1 - 0.5742 z_1^{-1} + 0.1012 z_2^{-1})(1 - 0.4646 z_1^{-1} + 0.05917 z_2^{-1})\]
Fig. 2. Low-pass digital filter designed in Example 1. (a) Contour plot of the magnitude responses (Contour values from origin: 0.9, 0.7, 0.5, 0.3, 0.1). (b) 3-D plot of the magnitude response. (c) 3-D plot of the phase response of the passband region ($-0.5\pi \leq \omega_1 \leq 0.5\pi$, $-0.5\pi \leq \omega_2 \leq 0.5\pi$).

Fig. 3. Band-pass digital filter designed in Example 2. (a) Contour plot of the magnitude responses (Contour values from origin: 0.1, 0.3, 0.5, 0.7, 0.9, 0.9, 0.7, 0.5, 0.3, 0.1). (b) 3-D plot of the magnitude response.
B. Example 2: Band-Pass Filter

A 20th-order 2-D circularly symmetric band-pass recursive digital filter with the following specifications is designed:

\[ H_D(z_1^{-1}, z_2^{-1}) = \begin{cases} 0.0, & \text{for } 0.0 \leq r \leq 0.1\pi \text{ rad/s} \\ 1.0, & \text{for } 0.3\pi < r < 0.5\pi \text{ rad/s} \\ 0.0, & \text{for } 0.7\pi \leq r \leq \pi \text{ rad/s} \end{cases} \]

where \( r = \sqrt{w_1^2 + w_2^2} \). The group delays in both \( w_1 \) and \( w_2 \) directions, \( \tau_1 \) and \( \tau_2 \), are required to be 2 up to the upper passband frequency where \( r \leq 0.5\pi \) rad/s. The resultant digital filter transfer function is obtained as

\[
H(z_1^{-1}, z_2^{-1}) = 2.6564E^{-10}
\]

\[
\begin{bmatrix}
1 \\
\hat{z}_1^{-1} \\
\hat{z}_2^{-1} \\
\hat{z}_3^{-1} \\
\hat{z}_4^{-1}
\end{bmatrix}^T
\begin{bmatrix}
1.0000 & 2.4000 & 3.4400 & 2.4000 & 1.0000 \\
2.4000 & -60.463 & -59.503 & -59.503 & 2.4000 \\
3.4400 & -59.503 & 752.01 & -59.503 & 3.4400 \\
2.4000 & -60.463 & -59.503 & -60.463 & 2.4000 \\
1.0000 & 2.4000 & 3.4400 & 2.4000 & 1.0000
\end{bmatrix}
\begin{bmatrix}
1 \\
\hat{z}_1^{-2} \\
\hat{z}_2^{-2} \\
\hat{z}_3^{-2} \\
\hat{z}_4^{-2}
\end{bmatrix}
\]

\[
\frac{(1 - 0.3178s_1^{-1} + 0.1380s_1^{-2})(1 - 0.2124s_2^{-1} + 0.1818s_2^{-2})(1 - 0.3178s_2^{-1} + 0.1380s_2^{-2})(1 - 0.2124s_1^{-1} + 0.1818s_1^{-2})}{(1 - 0.4003s_1^{-1} + 0.1130s_1^{-2})(1 - 0.4497s_2^{-1} + 0.0726s_2^{-2})(1 - 0.4003s_2^{-1} + 0.1130s_2^{-2})(1 - 0.4497s_1^{-1} + 0.0726s_1^{-2})}
\]

\[
\begin{bmatrix}
1 \\
\hat{z}_1^{-1} \\
\hat{z}_2^{-1} \\
\hat{z}_3^{-1} \\
\hat{z}_4^{-1}
\end{bmatrix}^T
\begin{bmatrix}
1.0000 & 2.4000 & 3.4400 & 2.4000 & 1.0000 \\
2.4000 & -7.9740 & -7.9740 & -7.9740 & 2.4000 \\
2.4000 & -7.9740 & -7.9740 & -7.9740 & 2.4000 \\
1.0000 & 2.4000 & 3.4400 & 2.4000 & 1.0000
\end{bmatrix}
\begin{bmatrix}
1 \\
\hat{z}_1^{-2} \\
\hat{z}_2^{-2} \\
\hat{z}_3^{-2} \\
\hat{z}_4^{-2}
\end{bmatrix}
\]

\[
\frac{(1 - 0.0495s_1^{-1} + 0.2604s_2^{-2})(1 - 0.2522s_1^{-1} + 0.4365s_2^{-2})(1 - 0.0495s_2^{-1} + 0.2604s_1^{-2})(1 - 0.2522s_2^{-1} + 0.4365s_1^{-2})}{(1 - 0.4338s_1^{-1} + 0.0962s_2^{-2})(1 - 0.5539s_2^{-1} + 0.0746s_1^{-2})(1 - 0.4338s_2^{-1} + 0.0962s_1^{-2})(1 - 0.5539s_1^{-1} + 0.0746s_2^{-2})}
\]

\[
\begin{bmatrix}
1 \\
\hat{z}_1^{-1} \\
\hat{z}_2^{-1} \\
\hat{z}_3^{-1} \\
\hat{z}_4^{-1}
\end{bmatrix}^T
\begin{bmatrix}
1.0000 & 2.4000 & 3.4400 & 2.4000 & 1.0000 \\
2.4000 & 10.341 & 11.301 & 10.341 & 2.4000 \\
2.4000 & 10.341 & 11.301 & 10.341 & 2.4000 \\
1.0000 & 2.4000 & 3.4400 & 2.4000 & 1.0000
\end{bmatrix}
\begin{bmatrix}
1 \\
\hat{z}_1^{-2} \\
\hat{z}_2^{-2} \\
\hat{z}_3^{-2} \\
\hat{z}_4^{-2}
\end{bmatrix}
\]

\[
\frac{(1 - 0.5742s_1^{-1} + 0.1012s_2^{-2})(1 - 0.4646s_1^{-1} + 0.0591s_2^{-2})(1 - 0.5742s_2^{-1} + 0.1012s_1^{-2})(1 - 0.4646s_2^{-1} + 0.0591s_1^{-2})}{(1 - 0.5742s_1^{-1} + 0.1012s_2^{-2})(1 - 0.4646s_1^{-1} + 0.0591s_2^{-2})(1 - 0.5742s_2^{-1} + 0.1012s_1^{-2})(1 - 0.4646s_2^{-1} + 0.0591s_1^{-2})}
\]

The magnitude responses of the filter is shown in Fig. 3(a)–(b). The phase response is identical to that of the Example 1. According to the given specifications of this example, the parameters of the RRMS error in group delay as given in [8, eq. (20)] are given by \( \tau = \tau_1 = \tau_2 = 2 \), \( M' = 6 \), \( L = 5 \), \( m = 2 \). By defining the group delay weighting functions, \( u_m \), to be all equal to 1, for \( g = 1 \, 6 \), the group delay RRMS error of this filter using the present design method is \( 1.03 \times 10^{-3} \). Similarly, the parameter of the RRMS error in magnitude response as given by [8, eq. (27)] is given by \( M'' = 66 \). By defining the magnitude response weighting functions, \( u_m \), to be all equal to 1 within the discrete frequency range as specified by [8, eq. (22)] or Table 1, the magnitude response RRMS error of this filter using the present method is 3.1. On designing a 20th-order filter with the same specifications and the same set of parameters as defined above using the method of [9], the corresponding group delay and magnitude response errors are, respectively, \( 2.9 \times 10^{-12} \) and 3.5.

IX. DESIGN OF NON-RECURSIVE DIGITAL FILTER USING OPTIMIZATION

It has been shown in Section VII that the non-recursive digital filters obtained by the modified McClellan transformation can be factorized into second- or fourth-order factors. In fact, even smaller modules for VLSI modular implementation can be obtained by expressing the 2-D non-recursive or numerator filter as...
cascade of first- and second-order sections. Hence

\[
N(z^{-1}, z^{-2}) = A(1 + a_{1}z^{-1})^{J}(1 + a_{0}z^{-1})^{J}
\]

\[
\cdot \begin{bmatrix}
1 & 1 & a_{1} & 1 \\
1 & b_{1} & a_{1} & 1 \\
1 & 1 & a_{1} & 1
\end{bmatrix}^{L}
\]

(14)

where \( J = 0 \) or 1, \( L \) is a positive integer such that \( J + 2L = 2M \leq N \). The unknowns \( a_{1}, b_{1}(i=1, 2, \ldots, L) \) of (14) can be determined by a nonlinear optimization algorithm such as the Fletcher–Powell algorithm [13] when combined with either (8) or (9) to meet a given set of magnitude and linear phase specifications. This provides an alternative approach to the design of the 2-D non-recursive filter.

X. CONCLUSIONS

In this paper, a new design method for 2-D linear phase circularly symmetric recursive digital filters has been presented. The proposed method was implemented on an IBM PC/AT (clock rate = 8 MHz) using Microsoft Fortran Compiler 3.3. The computational time for each example was about 2.3 min. Because of the speed of computation, it is suitable for the design of high-order 2-D recursive digital filters with high precision. The present method can be easily extended to the design of multidimensional linear phase spherically symmetric recursive digital filters. In summary, the major advantages of the present design method are: (a) simplicity in design methodology; (b) saving in computational time and improvement of convergence; (c) feasibility of VLSI modular implementation; (d) stability is guaranteed for all finite positive values of group delay; and (e) 2-D linear phase filter format.

REFERENCES


On Single Row Routing

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Abstract—Parallel algorithm for single row routing problem without backward moves and interstreet crossings is presented. The algorithm requires \( O(\log N \log \log N) \) time with \( N \) processors on a concurrent read concurrent write shared memory model or alternatively \( O(\log^{2} N) \) time with \( N \) processors on a concurrent read exclusive write shared memory model. The algorithm is then modified to run sequentially in \( O(N) \) time.

I. INTRODUCTION

The single row routing problem is as follows. Given \( N \) points \( V = \{1, 2, \ldots, N\} \) along a horizontal line \( L \) and a set of nets \( N_1, N_2, \ldots, N_m \) over \( V \) (each \( N_i \) is a subset of \( V \)) such that

i) \( N_i \cap N_j = \emptyset \), for \( i \neq j \).

ii) \( \bigcup_{i=1}^{m} N_i = V \).

The problem is to find a realization such that

i) there is a path of horizontal and vertical line segments between points of \( N_i \) so that all points of \( N_i \) are electrically equivalent;

ii) paths do not intersect each other.

The region above the line \( L \) is called the upper street and the region below this line the lower street. Horizontal segments in each street are to be laid in conductor paths, called tracks, which run parallel to the line \( L \). It is further required that a vertical line should not intersect any net more than once, i.e., backward moves are not permitted.

It is shown in [6] that in the absence of any other constraint the problem always has a solution. However the realization has interstreet crossovers, i.e., the paths in the realization cross from one street to another at a point not in \( V \). Raghavan and Sahni [3] have given an \( O(N^2) \) solution if no interstreet crossovers are to be permitted. They require \( O(n^2) \) time to determine feasibility and then \( O(N^2) \) time to obtain an optimal realization. The problem in absence of interstreet crossovers has the following additional constraint:

iii) nets do not cross from one street to another and a net either wholly lies in one street or a preassigned part of it is in one street and the remaining in the other.

For this problem a new sequential algorithm is proposed which runs in \( O(N) \) sequential time and is therefore better than Raghavan and Sahni's algorithm [3] for determining feasibility. The algorithm also finds a realization in \( O(N) \) time.

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