The corrected form of the lemma of [1] is

**Lemma.** Given $A$, $b$, and $c$ as in (3) such that the pair $(c', A)$ is completely observable (i.e., the vectors $c, Ac, \ldots, (A)^{2m-1}c$ are linearly independent) and given that (2) is satisfied, then there exists a symmetric, positive definite matrix $H$, a vector $q$, and a scalar $\gamma \neq 0$ satisfying (1).

**Proof:** The left-hand side of (2) can be expressed as [1]
\[
\delta + 2\text{Re}\left\{c'(e^{i\omega}I - A)^{-1}b\right\} = \frac{|\theta(e^{i\omega})|^2}{|e^{im\psi}(e^{i\omega})|^2}
\]
where $\theta$ is defined in [1]. The present $(A, b)$, see (3), being in phase variable form is already completely controllable [3, p. 241]. This fact and the assumption that $(c', A)$ is completely observable assure that there is no common factor between $\theta(z)$ and $z^m\psi(z)$, $\gamma \neq 0$ and $q$ can now be defined by requiring that [1]
\[
\gamma - q'(zd - I)^{-1}b = \frac{\theta(z)}{z^m\psi(z)}.
\]

Since there is no common factor between $\theta(z)$ and $z^m\psi(z)$ and since $(A, b)$ is completely controllable, $(q', A)$ in (6) must be completely observable. This complete observability of $(q', A)$ assures [5, p. 483] the positive definiteness of $H$ in (1a).

The remaining steps of the proof are as those in [1]. Thus the lemma of [1] is perfectly valid if it is assumed that $(c', A)$ is completely observable. Note that, in the example of [2], $(c', A)$ is not completely observable.

### III. Discussion

As noted above, the complete observability of $(c', A)$ leads to the complete observability of $(q', A)$. The complete observability of $(q', A)$ apart from assuring the positive definiteness of $H$ serves an additional purpose. Recall the expression for $V(n + 1) - V(n)$ [1]:
\[
V(n + 1) - V(n) = - \{q's(n) - \gamma e(n)\}^2 + \sum_{r=1}^{m} a_r e(n)\{x(n) - x(n - r)\} + \sum_{r=1}^{m} \beta_r e(n)\{x(n) + x(n - r)\}.
\]

It is noted that [1]
\[
V(n + 1) - V(n) = 0 \quad \text{(which implies } e(n) = 0 \text{)}
\]
requires
\[
q's(n) = 0
\]
and correspondingly, the system reduces to
\[
s(n) = A's(n - 1).
\]

Since $(q', A)$ is completely observable, the only trajectory of (10) for which (9) holds true is $s(n) = 0$ [3, p. 94].

Thus though $V(n + 1) - V(n)$ is only negative semidefinite, the asymptotic stability of the null solution is assured by the observability of $(q', A)$.

**References**


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A Multi-Output Wave Digital Biquad Using Magnitude Truncation Instead of Controlled Rounding

HON KEUNG KWAN

**Abstract** — The use of magnitude truncation instead of controlled rounding for the elimination of zero-input and constant-input oscillations in the wave digital biquad derived from the feedforward RC-active configuration is described. We also describe how the structure could be used for the simultaneous realization of all types of second-order digital filters.

**I. Introduction**

The problem of eliminating zero-input and constant-input oscillations in second-order digital filters has received considerable attention. Some recent references could be found in [1]–[7]. There are two general approaches to deal with this problem. One is to apply magnitude truncation [1]–[4], and the other is to apply controlled rounding [5], [7], to the signals at the two outputs of a recursive two-port.

Using controlled rounding, the input and the recursive part of a wave digital biquad without zero-input and constant-input oscillations could be derived from the digital transformation of the feedforward three-amplifier RC-active biquad [7]. In the hardware implementation of the controlled rounding arithmetic, the quantization of a signal requires a comparison with its controlled or steady state signal. This results in an increase in the hardware complexity as compared to the magnitude truncation of the signal.

In this contribution, we describe in Section II how magnitude truncation can be used instead of controlled rounding for the elimination of both zero-input and constant-input oscillations. Moreover, with appropriate output connections, the filter structure could be used for the simultaneous realization of all types of second-order digital filters. This is presented in Section III.

**II. Magnitude Truncation Instead of Controlled Rounding**

From the fig. 3 of [7], we obtain the input and the recursive part of the wave digital biquad which has been shown to be free

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Fig. 1. The input and recursive part of the wave digital filter structure using controlled rounding for the quantization of $y_1(k)$ and $y_2(k)$.

Fig. 2. The input and recursive part of the wave digital filter structure using magnitude truncation (MT) instead of controlled rounding.

from both zero-input and constant-input oscillations by using controlled rounding arithmetic. The signal flow graph of the digital structure is shown in Fig. 1. From Fig. 1, we obtain

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} -a_1 & a_1 -1 & -a_1 \\ -a_2 & a_2 & 1 - a_2 \end{bmatrix} \begin{bmatrix} u(k) \\ x_1(k) \\ x_2(k) \end{bmatrix}.$$  \hspace{1cm} (1)

Under constant-input condition, we have $u(k) = u$, where $u$ is a constant. Without quantization, under constant-input and steady state (as $k$ tends to $\infty$) conditions, $y_i(k) = \text{constant}$ for $i=1,2$. The latter implies that $x_i(k) = y_i(k) = \text{constant}$ for $i=1,2$. On substituting $u(k) = u$ and $x_i(k) = y_i(k)$ for $i=1,2$ into (1), we obtain

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} -a_1 & a_1 -1 & -a_1 \\ -a_2 & a_2 & 1 - a_2 \end{bmatrix} \begin{bmatrix} u \\ y_1(k) \\ y_2(k) \end{bmatrix}.$$  \hspace{1cm} (2)

Solving (2), the steady state responses at $y_1(k)$ and $y_2(k)$ are

$$y_1(k) = 0 \quad (3)$$

$$y_2(k) = -u \quad (4)$$

According to the controlled rounding arithmetic [5], if constant-input oscillations have to be eliminated from the filter structure, $y_1(k)$ has to be rounded towards its steady-state value (i.e., $0$) and $y_2(k)$ has to be rounded towards its steady state value (i.e., $-u$). The latter operation requires

$$\lfloor y_2(k) \rfloor_{\text{CR}} = (-u) \lfloor y_2(k) - (-u) \rfloor \quad (5)$$

where $\lfloor y_2(k) \rfloor_{\text{CR}}$ represents the value of $y_2(k)$ after controlled rounding. In other words, $y_2(k)$ has to be rounded upwards if $-u > y_2(k)$ and downwards if $-u < y_2(k)$. This operation requires the comparison of two signals, $y_2(k)$ and $-u$, which results in an increase of hardware complexity. In practice, controlled rounding of $y_2(k)$ can be achieved by subtracting and then adding, respectively, the steady state value $-u$ to $y_2(k)$ immediately before and after magnitude truncation. From (3), the steady state value of $y_1(k)$ is $0$, hence, controlled rounding of $y_1(k)$ can be achieved by magnitude truncation of $y_1(k)$. Consequently, magnitude truncation can be used instead of controlled rounding for the elimination of constant-input oscillations. The resultant digital structure which employs such magnitude truncation instead of controlled rounding can be obtained from Fig. 1 and is shown in Fig. 2. After rearranging Fig. 2 into the form shown in Fig. 3, it can be seen easily that the resultant digital structure shown in Fig. 3 conforms to a particular form of the zero-input and constant-input stable digital filter structure shown in Fig. 2 of [2] (where $f_1 = 0$ and $f_2 = -1$).

III. MULTI-OUTPUT DIGITAL FILTER STRUCTURE

With appropriate output connections, a multi-output second-order digital filter can be obtained (from Fig. 3) which could be used for the simultaneous realization of low-pass (LP), high-pass (HP), band-pass (BP), and band-stop (BS) Butterworth, Chebyshev, and elliptic digital filters, and all-pass (AP) digital filters. The resultant multi-output digital filter is shown in Fig. 4. The transfer function between each output and the input can be obtained by taking the $z$-transform of each output over the $z$-transform of the input. They are summarized as follows.

$$T_{LP}(z^{-1}) = \frac{Z\left[ y_{LP}(k) \right]}{Z\left[ u(k) \right]} = -a_2 \frac{(1 + z^{-1})^2}{D(z^{-1})} \quad (6)$$

$$T_{HP}(z^{-1}) = \frac{Z\left[ y_{HP}(k) \right]}{Z\left[ u(k) \right]} = a_1 \frac{(1 - z^{-1})^2}{D(z^{-1})} \quad (7)$$

$$T_{BP}(z^{-1}) = \frac{Z\left[ y_{BP}(k) \right]}{Z\left[ u(k) \right]} = - \frac{1 - z^{-2}}{D(z^{-1})} \quad (8)$$

$$T_{BS}(z^{-1}) = \frac{Z\left[ y_{BS}(k) \right]}{Z\left[ u(k) \right]} = \frac{1 - 2 \left( \frac{a_1 + \beta a_2}{a_1 - \beta a_2} \right) z^{-1} + z^{-2}}{D(z^{-1})} \quad (9)$$
\[
T_{sr}(z^{-1}) = \frac{Z[v_{sr}(k)]}{Z[u(k)]} = \frac{(a_1 + a_2 - 1) - (a_1 - a_2)z^{-1} + z^{-2}}{D(z^{-1})}
\]

where
\[
D(z^{-1}) = 1 - (a_1 - a_2)z^{-1} + (a_1 + a_2 - 1)z^{-2}
\]

and \(Z[\cdot]\) represents the \(z\)-transform of \([\cdot]\).

### IV. CONCLUSIONS

The use of magnitude truncation instead of controlled rounding for the elimination of zero-input and constant-input oscillations in the wave digital biquad advanced in [7] has been described. This reduces the level of complexity for hardware implementation. In fact, the resultant digital structure represents one of the possible forms of the general digital filter structure suggested in [2] for the elimination of both zero-input and constant-input oscillations in second-order passive digital filters. Moreover, the structure has been used to form a multi-output digital filter for the realization of all types of second-order digital filters.

### REFERENCES


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**The Time-Domain Response of Minimum Phase Networks**

**R R CLARKE**

Abstract—It is shown that the pulse response of any minimum-phase network can be viewed as comprising a main pulse followed, but not preceded, by a series of echoes. The trailing echo performance gives insight into several, hitherto unexplained, empirical observations concerning the time domain response of minimum-phase networks. These include the absence of precursor ringing in the impulse and step responses and the presence of more distortion following, rather than preceding, the main lobe peak. In addition, for pulses with equally spaced precursor zero crossings, it is shown that minimum-phase networks approximately preserve this behavior. The result confirms the suitability of decision feedback equalizers for digital subscriber access to an integrated services digital network via existing copper pair cable.

I. INTRODUCTION

In the past, the pulse response of minimum-phase networks was of interest primarily in the context of filters (e.g., [1]). Now, this subject has received renewed interest with the intense research into digital subscriber access to the Integrated Services Digital Network (ISDN) via existing two-wire subscriber loops (e.g., [2]).

Several related empirical observations concerning the time-domain response of minimum-phase networks are described in the literature. One observation is the impulse and step responses of a minimum-phase network exhibits no precursor ringing (e.g., [3]). The validity of this observation is confirmed by examining the impulse and step responses of minimum-phase filters (e.g., [1]). Another observation is that the pulse response exhibits more distortion following, rather than preceding, the main lobe peak (e.g., [4]). Another related observation is that when minimum-phase networks are excited by pulses possessing a precursor with equally spaced zero crossings, then the response exhibits near equally spaced zero crossings (e.g., [2]). The last two observations can be seen to be valid in the pulse response contained in Fig. 1. This is the response of 3.0 km of 0.4 mm diameter copper pair cable to a 20-percent raised-cosine pulse shape with a bit period corresponding to a transmission rate of 144 kbit/s.

To the author’s knowledge, no explanation for these observations has been offered in the literature. This is remedied in this paper by providing a theoretical understanding of the nature of the time-domain response of minimum-phase networks. It is shown that the pulse response of a minimum-phase network can be decomposed into an ideal pulse followed by a series of delayed replicas (echoes) of this pulse. This fact is used to explain the previous observations. The approach of this letter is an extension of [5] and [6].

II. A MINIMUM-PHASE NETWORK TRANSFER FUNCTION

For a minimum-phase network, the attenuation and phase (or group delay) responses are related via the Hilbert transform (e.g.,