A multi-output first-order digital filter structure has been presented. Preliminary results indicate that zero-input, constant-input, and forced overflow oscillations can be eliminated in this first-order structure and in the second-order structure suggested in [1]. Due to these oscillation-free properties, together with their simplicities in structure, they are attractive for VLSI implementation.

A multi-output second-order digital filter without limit cycle oscillations

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Abstract — A multi-output second-order digital filter without zero-input, constant-input, and forced overflow oscillations is presented. The filter structure can be used for the simultaneous realization of low-pass, high-pass, band-pass, band-stop, all-pass, and general biquadratic transfer functions using two delays, four multipliers, and eleven adders.

I. INTRODUCTION

Due to finite arithmetic operations, limit-cycle oscillations can occur in digital filters under zero-input, constant-input, and forced overflow conditions. Recently, we have investigated the design of second-order digital filters without zero-input, constant-input, and forced overflow oscillations [1]-[3]. Meanwhile, considerable work has been reported in [4]-[6] on related second-order digital filter structures. A comparison of our work with those of their work reveals that our filter structures are derived from passive gyrator circuits whereas their filter structures are derived from RC active configurations. Moreover, our multi-output filter structures are slightly less complicated than their multi-output filter structures. In this contribution, we further illustrate our design method using the third digital two-port example as given in table I of [2]. In a pattern similar to those given in [3] and [5], we also show how the filter structure could be used for the simultaneous realization of all types of second-order digital transfer functions.

II. FINITE WORDLENGTH STABILITY

With reference to [1] and [2], the zero-input stability for the input and recursive part of the filter shown in Fig. 1 can be obtained. Under zero-input conditions, the state equations can be expressed as

\[
\begin{align*}
\begin{bmatrix}
    x_1(k+1) \\
    x_2(k+1)
\end{bmatrix}
&= \begin{bmatrix}
    \alpha_1 & 1 - \alpha_2 \\
    -(1 + \alpha_1) & \alpha_2
\end{bmatrix}
\begin{bmatrix}
    x_1(k) \\
    x_2(k)
\end{bmatrix}
\end{align*}
\]

where \([ \cdot ]_{MT}\) represents the operation of magnitude truncation. Assuming the filter structure is zero-input stable, we have

\[
\begin{bmatrix}
    x_1(k+1) \\
    x_2(k+1)
\end{bmatrix}
\]

Under constant-input conditions,

\[
u(k) = u(k+1) = u, \quad \text{for } k \geq 0.
\]

Again, from Fig. 1, we obtain

\[
\begin{bmatrix}
    x_1(k+1) \\
    x_2(k+1)
\end{bmatrix}
= \begin{bmatrix}
    \alpha_1 & 1 - \alpha_2 \\
    -(1 + \alpha_1) & \alpha_2
\end{bmatrix}
\begin{bmatrix}
    x_1(k) \\
    x_2(k)
\end{bmatrix}
\]

From (3) and (4), we obtain

\[
\begin{bmatrix}
    x_1(k+1) \\
    x_2(k+1)
\end{bmatrix}
= \begin{bmatrix}
    \alpha_1 & 1 - \alpha_2 \\
    -(1 + \alpha_1) & \alpha_2
\end{bmatrix}
\begin{bmatrix}
    x_1(k) \\
    x_2(k)
\end{bmatrix}_{MT}
\]

where

\[
\begin{bmatrix}
    x_1(k) \\
    x_2(k)
\end{bmatrix} = \begin{bmatrix}
    x_1(k) \\
    x_2(k) - u
\end{bmatrix}
\]

Equation (5) is equivalent to (1) except for a change of state variables. The relationship between the new and old state variables is given by (6). Thus the filter shown in Fig. 1 is constant-input stable if it is zero-input stable. Consequently, we have

\[
\lim_{k \to \infty} \begin{bmatrix}
    x_1(k) \\
    x_2(k)
\end{bmatrix} = \begin{bmatrix}
    0 \\
    0
\end{bmatrix}
\]
Fig. 2. A multi-output second-order digital filter \((\beta_1 - (a_1 - a_2)/\omega_2 - 1)\) for \(v_{AP}(k)\).

In practice, by using extra wordlength for overflow prevention at appropriate points of the filter, together with magnitude truncation at the outputs of the digital two-port, zero-input, and constant-input oscillations can be eliminated. Moreover, if the saturation characteristics suggested in [7] is also observed, forced overflow oscillations can also be eliminated.

\[
T_{AP}(z^{-1}) = \frac{Z[v_{AP}(k)]}{Z[u(k)]}
= \frac{(a_1 - a_2 + 1)(a_2 + a_2) z^{-1} + z^{-2}}{D(z^{-1})}
\]

where

\[
D(z^{-1}) = (a_1 + a_2) z^{-1} + (a_1 - a_2 + 1) z^{-2}
\]

and \(Z[\cdot]\) represents the z-transform of \([\cdot]\).

### III. Multi-Output Second-Order Digital Filter

A multi-output second-order digital filter which is capable of realizing low-pass (LP), high-pass (HP), band-pass (BP), band-stop (BS), all-pass (AP), and general biquadratic (G) digital transfer functions is shown in Fig. 2. The transfer functions can be expressed as follows:

\[
T_{LP}(z^{-1}) = \frac{Z[v_{LP}(k)]}{Z[u(k)]}
= (1 - a_2) \cdot \left(1 + z^{-1}\right)^2
\]

\[
T_{HP}(z^{-1}) = \frac{Z[v_{HP}(k)]}{Z[u(k)]}
= (1 - a_2) \cdot \left(1 - z^{-1}\right)^2
\]

\[
T_{BP}(z^{-1}) = \frac{Z[v_{BP}(k)]}{Z[u(k)]}
= (a_2 - 1) \cdot \left(1 - z^{-2}\right) / D(z^{-1})
\]

\[
T_{BS}(z^{-1}) = \frac{Z[v_{BS}(k)]}{Z[u(k)]}
= (1 - a_2)(\beta_1 + 1)
\]

\[
1 + 2 \frac{\beta_1 - 1}{\beta_1 + 1} z^{-1} + z^{-2} / D(z^{-1})
\]

### IV. Conclusions

A multi-output second-order digital filter structure has been presented. It has been shown to be constant-input and forced overflow stable provided it is zero-input stable. The filter structure is flexible and can be used for the simultaneous realization of all types of second-order transfer functions. The quantized pole locations of the present filter structure shown in Fig. 2 (with \(a_1\) and \(a_2\) quantized to \(2^{-6}\)) is identical to that as given in fig. 3 of [8].

### REFERENCES


